All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Graph the following inequality on the number line provided.

$$9x + 2 < 2x + 79$$

2. Graph the following inequality on the number line provided.

$$x^5 \geq 256x$$

3. Graph the following inequality on the number line provided.

$$|x^2 - 3x - 55| \geq 15$$

ANSWERS

(1 pt) 1.

(2 pts) 2.

(3 pts) 3.

All answers must be in simplest exact form in the answer section.

NO CALCULATORS ALLOWED

1. Solve for x.

$$(x + 3)(x + 4) = x^2 + 68$$

2. Simplify the following expression to a single polynomial.

$$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

3. Calculate

$$\frac{45^3 + 34^3}{(45 - 34)^2 + 45 \cdot 34} + \frac{23^3 - 12^3}{(23 + 12)^2 - 23 \cdot 12}$$

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Worcester County Mathematics League

Junior Varsity Meet 3 - March 26, 2025

Round 3 - Techniques of Counting and Probability

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Jacob rolls a fair 13-sided die with outcomes ranging from 1 to 13. What is the probability that the number rolled is a multiple of 3?

2. Coach Jasper has a group of 6 candidates and needs to select a subset of them to form a team. From the chosen team members, he then selects a subset to be captains. It is possible for the team to be empty or for no captains to be chosen. How many different ways can Coach Jasper form a team and assign captains?

3. Ron, Adi, and Leonard each choose 2 different colors from a set of 7 possible colors to paint their bedrooms. Ron picks first. Adi picks next and can choose any 2 colors, including those chosen by Ron. Leonard goes last but must select 2 colors that are completely different from any of Ron's and Adi's choices. How many possible ways can they make their selections?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

All answers must be in simplest exact form in the answer section.

NO CALCULATORS ALLOWED

1. A cylinder has volume 144π. If its height is 9, what is the radius of its base?

2. A right triangle has an area of 96. One of its legs is 4 units longer than the other. What is the length of the hypotenuse?

3. A sphere of radius 2 is inscribed in a cone whose base has radius 4. What is the volume of the cone?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Solve for x.

$$3(14x + 1) = 5(9x + 26)$$

2. Evaluate the following expression.

$$\frac{64^{-\frac{2}{3}} \cdot 16^{\frac{3}{2}}}{2^{-3}}$$

3. Compute

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{16} + \frac{9}{32} + \ldots$$

4. Find the smallest positive integer that is divisible by 11 and whose digits sum to 22.

5. How many integers between 1 and 500, inclusive, are not divisible by 12 or 16?

6. A robot moves on a 5 by 5 grid starting at the bottom-left corner, $(0,0)$. It can only move right or up by exactly one unit at each step. How many different paths can the robot take to reach the top-right corner, $(5,5)$, without passing through $(2,4)$?

7. Solve for x.

$$\frac{1}{x+1} + \frac{1}{x+7} = \frac{1}{4}$$

8. Two identical circles of radius 1 are externally tangent at point O. Both are internally tangent to a larger circle centered at O. What is the length of a chord of the larger circle, not passing through O, that is tangent to both of the smaller circles?

ANSWERS

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

Worcester County Mathematics League
Junior Varsity Meet 3 - March 26, 2025
Answer Key

Round 1 - Evaluation and Order of Operations

1. $x < 11$

2. $-4 \leq x \leq 0, x \geq 4$

3. $x \leq -7, -5 \leq x \leq 8, x \geq 10$

Round 2 - Operations on Polynomials

1. 8

2. $x^3 + y^3 + z^3 - 3xyz$

3. 90

Round 3 - Fractions...

1. $\frac{4}{13}$

2. 729

3. 2100

Round 4 - Perimeter, Area, and Volume

1. 4

2. 20

3. $\frac{256\pi}{9}$

Team Round

1. $-\frac{127}{3}$

2. 32

3. 3

4. 2299

5. 438

6. 192

7. ± 5

8. $2\sqrt{3}$

School: _____

Circle your team below.

Team #1   Team #2   Team #3   Team #4   Team #5

Team #6   Team #7   Team #8   Team #9   Team #10

Name: _____

For Official Use Only
Score:

Reminders:

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- All electronic devices should be OFF and not in plain sight.

- Calculators are **not** allowed during this round.

- Do not turn this paper over until the moderator says, "Begin!"

WOCOMAL Junior Varsity Meet 3
Round 1: Graphing on a Number Line

WOCOMAL Junior Varsity Meet 3
Round 2: Operations on Polynomials

School: _____

Name: _____

Reminders:

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WOCOMAL Junior Varsity Meet 3
Round 3: Techniques of Counting and Probability

School: _____

Name: _____

For Official Use Only
Score:

WOCOMAL Junior Varsity Meet 3
Round 4: Perimeter, Area, and Volume

Worcester County Mathematics League

Junior Varsity Meet 3 - March 26, 2025

TEAM ROUND

School Name: _____

Team #: _____

Team Members:

_____

_____

_____

_____

_____

Score = _____ / 16

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Graph the following inequality on the number line provided.

$$9x + 2 < 2x + 79$$

> **Solution:** We get
> $$7x < 77$$
> $$\Rightarrow \boxed{x < 11}$$

2. Graph the following inequality on the number line provided.

$$x^5 \geq 256x$$

> **Solution:** We have
> $$\Rightarrow x^5 - 256x \geq 0$$
> $$\Rightarrow x(x^4 - 256) \geq 0$$
> Thus, if $x < 0$, we want $x^4 - 256 \leq 0 \Rightarrow -4 \leq x \leq 4$, and if $x \geq 0$, we want $x \leq -4$ or $x \geq 4$.
> Thus, in total we have $\boxed{-4 \leq x \leq 0, x \geq 4}$.

3. Graph the following inequality on the number line provided.

$$|x^2 - 3x - 55| \geq 15$$

Solution: If the value in the absolute value is positive, we want

$$x^2 - 3x - 55 \geq 15$$
$$\Rightarrow x^2 - 3x - 70 \geq 0$$
$$\Rightarrow (x + 7)(x - 10) \geq 0$$

This gives $x \leq -7, x \geq 10$.

If the value in the absolute value is negative, we want

$$x^2 - 3x - 55 \leq -15$$
$$\Rightarrow x^2 - 3x - 40 \leq 0$$
$$\Rightarrow (x + 5)(x - 8) \leq 0$$

This gives $-5 \leq x \leq 8$.

In total, we have $\boxed{x \leq -7, -5 \leq x \leq 8, x \geq 10}$.

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Solve for x

$$(x+3)(x+4) = x^2 + 68$$

> **Solution:** We have
>
> $$\Rightarrow x^2 + 7x + 12 = x^2 + 68$$
> $$\Rightarrow 7x = 56$$
> $$\Rightarrow x = \boxed{8}$$

2. Simplify the following expression to a single polynomial.

$$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

> **Solution:** We have that each $x \cdot y^2$ is canceled out by $y \cdot (-xy)$, so we are left with just
>
> $$\boxed{x^3 + y^3 + z^3 - 3xyz}$$

3. Calculate

$$\frac{45^3 + 34^3}{(45 - 34)^2 + 45 \cdot 34} + \frac{23^3 - 12^3}{(23 + 12)^2 - 23 \cdot 12}$$

Solution: We have $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ and $(x-y)^2 + xy = x^2 - xy + y^2$. Thus,

$$\frac{x^3 + y^3}{(x-y)^2 + xy} = x + y$$

Similarly,

$$\frac{x^3 - y^3}{(x+y)^2 - xy} = x - y$$

Thus, the expression equals

$$(45 + 34) + (23 - 12) = \boxed{90}$$

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Jacob rolls a fair 13-sided die with outcomes ranging from 1 to 13. What is the probability that the number rolled is a multiple of 3?

   Solution: There are 4 multiples of 3 between 1 and 13, so the probability is $\boxed{\dfrac{4}{13}}$

2. Coach Jasper has a group of 6 candidates and needs to select a subset of them to form a team. From the chosen team members, he then selects a subset to be captains. It is possible for the team to be empty or for no captains to be chosen. How many different ways can Coach Jasper form a team and assign captains?

   Solution: Each candidate can either be not selected, selected to be on the team but not a captain, or selected to be a captain. Thus, there are $3^6 = \boxed{729}$ ways Coach Jasper can form a team and assign captains.

3. Ron, Adi, and Leonard each choose 2 different colors from a set of 7 possible colors to paint their bedrooms. Ron picks first. Adi picks next and can choose any 2 colors, including those chosen by Ron. Leonard goes last but must select 2 colors that are completely different from any of Ron's and Adi's choices. How many possible ways can they make their selections?

   Solution: Leonard can select his colors in $\binom{7}{2} = 21$ ways. Then, each of Ron and Adi can pick their colors in $\binom{5}{2} = 10$ ways. Thus, they can make their selections in $21 \cdot 10 \cdot 10 = \boxed{2100}$ ways.

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. A cylinder has volume 144π. If its height is 9, what is the radius of its base?

> **Solution:** The area of the base is $144\pi/9 = 16\pi$. Then, the radius of the base is $\sqrt{16} = \boxed{4}$.

2. A right triangle has an area of 96. One of its legs is 4 units longer than the other. What is the length of the hypotenuse?

> **Solution:** If x is the lenght of the shorter leg, we have
>
> $$\frac{x(x+4)}{2} = 96$$
> $$\Rightarrow x(x+4) = 192$$
>
> Solving, we get the legs are 12 and 16, so the hypotenuse is $\sqrt{12^2 + 16^2} = \boxed{20}$.

3. A sphere of radius 2 is inscribed in a cone whose base has radius 4. What is the volume of the cone?

> **Solution:** Let A be the apex of the cone, O be the center of the sphere, C be the center of the base, B be a point on the circumference of the base, X be the point on AB where the sphere touches the cone. We have that $\triangle AXO \sim \triangle ACB$ since they have the same angle at A and both have a right angle. Thus,
>
> $$\frac{XO}{AO} = \frac{BC}{AB}$$

$$\Rightarrow 2AB = 4(AC - 2)$$
$$\Rightarrow AB = 2AC - 4$$

Also,

$$\frac{XO}{AX} = \frac{BC}{AC}$$
$$\Rightarrow 2AC = 4(AB - 4)$$
$$\Rightarrow AC = 2(AB - 4)$$
$$\Rightarrow AC = 2(2AC - 8)$$
$$\Rightarrow AC = 4AC - 16$$
$$\Rightarrow AC = \frac{16}{3}$$

Then, the volume is $\frac{1}{3}\frac{16}{3}4^2\pi = \boxed{\dfrac{256\pi}{9}}$.

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Solve for x.

$$3(14x + 1) = 5(9x + 26)$$

Solution: We get

$$\Rightarrow 42x + 3 = 45x + 130$$
$$\Rightarrow 3x = -127$$
$$\Rightarrow x = \boxed{-\frac{127}{3}}$$

2. Evaluate the following expression.

$$\frac{64^{-\frac{2}{3}} \cdot 16^{\frac{3}{2}}}{2^{-3}}$$

Solution: We get
$$= 2^{6(-2/3)+4(3/2)+3)} = 2^{-4+6+3} = 2^5 = \boxed{32}$$

3. Compute

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{16} + \frac{9}{32} + \dots$$

Solution: If

$$x = \frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{16} + \frac{9}{32} + \ldots$$

$$\Rightarrow \frac{1}{2}x = \frac{1}{4} + \frac{3}{8} + \frac{5}{16} + \frac{7}{32} + \frac{9}{64} + \ldots$$

$$\Rightarrow \frac{1}{2}x = 1 - \frac{1}{2}x = \frac{1}{2} + \frac{2}{4} + \frac{2}{8} + \frac{2}{16} + \ldots$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

$$\Rightarrow x = \boxed{3}$$

4. Find the smallest positive integer that is divisible by 11 and whose digits sum to 22.

Solution: Two digits aren't enough to sum to 22, so the answer is at least 3 digits. If it is 3 digits, divisibility rules say that the first and last digit must sum to the middle digit or 11 more than the middle digit. If the first and last digits sum to the middle digit, the sum is just twice the middle digit which cannot be 22. If the first and last digits sum to 11 more than the middle digit, the total sum is odd. Thus, the answer is at least 4 digits. The first and third digit's sum must be equal to, 11 more, or 11 less than the second and fourth digit's sum. If it is 11 more or less the total sum is odd, so the sums must be equal. To minimize the digits with higher place value, we get the number $\boxed{2299}$.

5. How many integers between 1 and 500, inclusive, are not divisible by 12 or 16?

Solution: There are $\lfloor 500/12 \rfloor = 41$ numbers divisible by 12, $\lfloor 500/16 \rfloor = 31$ numbers divisible by 12, and $\lfloor 500/48 \rfloor = 10$ numbers divisible by both. By the principle of inclusion-exclusion, there are $500 - 41 - 31 + 10 = \boxed{438}$ numbers not divisible by 12 or 16.

6. A robot moves on a 5 by 5 grid starting at the bottom-left corner, $(0, 0)$. It can only move right or up by exactly one unit at each step. How many different paths can the robot take to reach the top-right corner, $(5, 5)$, without passing through $(2, 4)$?

> **Solution:** There are $\binom{10}{5} = 252$ paths going from $(0,0)$ to $(5,5)$. There are $\binom{6}{2}\binom{4}{3} = 60$ paths that go through $(2,4)$. Thus, the number of paths that do not pass through $(2,4)$ is $252 - 60 = \boxed{192}$.

7. Solve for x.

$$\frac{1}{x+1} + \frac{1}{x+7} = \frac{1}{4}$$

> **Solution:** Multiplying both sides by $4(x+1)(x+7)$, we get
>
> $$4x + 28 + 4x + 4 = x^2 + 8x + 7$$
> $$\Rightarrow x^2 - 25 = 0$$
> $$\Rightarrow x = \boxed{\pm 5}$$

8. Two identical circles of radius 1 are externally tangent at point O. Both are internally tangent to a larger circle centered at O. What is the length of a chord of the larger circle, not passing through O, that is tangent to both of the smaller circles?

> **Solution:** The radius of the larger circle is 2. The distance from O to the chord must be 1. Then, by the Pythagorean Theorem, half of the chord must be $\sqrt{3}$, so the total length of the chord is $\boxed{2\sqrt{3}}$.

ANSWERS

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

Round 1 - Evaluation and Order of Operations

1. $x < 11$

2. $-4 \leq x \leq 0, x \geq 4$

3. $x \leq -7, -5 \leq x \leq 8, x \geq 10$

Round 2 - Operations on Polynomials

1. 8

2. $x^3 + y^3 + z^3 - 3xyz$

3. 90

Round 3 - Fractions...

1. $\frac{4}{13}$

2. 729

3. 2100

Round 4 - Perimeter, Area, and Volume

1. 4

2. 20

3. $\frac{256\pi}{9}$

Team Round

1. $-\frac{127}{3}$

2. 32

3. 3

4. 2299

5. 438

6. 192

7. ± 5

8. $2\sqrt{3}$

School: _____

Circle your team below.

Team #1   Team #2   Team #3   Team #4   Team #5

Team #6   Team #7   Team #8   Team #9   Team #10

Name: _____

> **For Official Use Only**
> **Score:**

YES   **Western Mass ARML Member?**   NO
(circle one)

Reminders:

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- Do not turn this paper over until the moderator says, "Begin!"

WOCOMAL Junior Varsity Meet 3
Round 1: Graphing on a Number Line

School: _____

Circle your team below.

Team #1   Team #2   Team #3   Team #4   Team #5

Team #6   Team #7   Team #8   Team #9   Team #10

Name: _____

For Official Use Only

Score:

YES   Western Mass ARML Member?   NO
(circle one)

Reminders:

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WOCOMAL Junior Varsity Meet 3
Round 2: Operations on Polynomials

School: —————————————————————————————————

Name: —————————————————————————————————

Reminders:

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WOCOMAL Junior Varsity Meet 3
Round 3: Techniques of Counting and Probability

School: _____

Name: _____

WOCOMAL Junior Varsity Meet 3
Round 4: Perimeter, Area, and Volume

Worcester County Mathematics League

Junior Varsity Meet 3 - March 26, 2025

TEAM ROUND

School Name: _____

Team #: _____

Team Members:

_____

_____

_____

_____

_____

Score = _____ / 16